# Seleucid, Demotic and Mediterranean Mathematics versus Chapters VIII and IX of the *Nine Chapters*: Accidental or Significant Similarities?*

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**Abstract**　Similarities of geometrical diagrams and arithmetical structures of problems have often been taken as evidence of transmission of mathematical knowledge or techniques between China and "the West". Confronting on one hand some problems from Chapter VIII of the *Nine Chapters* with comparable problems known from Ancient Greek sources, on the other a Seleucid collection of problems about rectangles with a subset of the triangle problems from Chapter IX, it is concluded（1）that transmission of some arithmetical riddles without method – not "from Greece" but from a transnational community of traders – is almost certain, and that these inspired the Chinese creation of the *fangcheng* method, for which Chapter VIII is a coherent presentation;（2）that transmission of the geometrical problems is to the contrary unlikely, with one possible exception, and that the coherent presentation in Chapter IX is based on local geometrical practice.

**Keywords**　*Nine Chapters on Arithmetic*, *Fangcheng* method, transmission, mathematical riddles, Babylonian geometry

Yvonne Dold – Samplonius

*in memoriam*

## *Two pictures and a winged transmission*

First of all, as introduction, two pictures.

To the left, the decoration of the "Mathematics and Systems Science" tower in the campus of the Chinese Academy of Sciences in Beijing, borrowed from Zhao Shuang's third-century CE
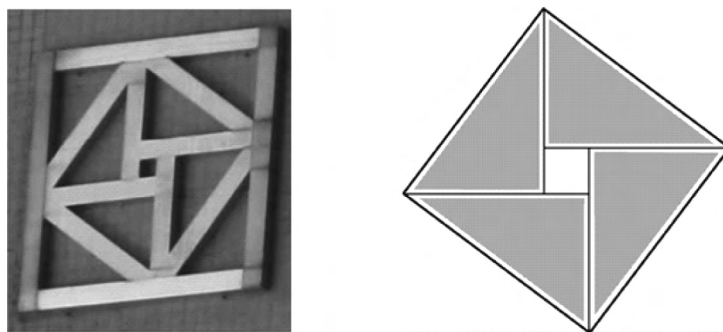
Fig. 1

commentary to the *Gnomon of the Zhou* – cf. （Cullen 1996: 206; Chemla & Guo 2004: 695–701）; to the right my own diagram reconstructed from the description of the procedure of the Old Babylonian problem $Db_2$–146 （c. 1775 BCE） in which the sides of a rectangle are to be determined from its area and the diagonal （Høyrup 2002a: 257–259）. When I first made it it should be added I did not know about the Chinese diagram.

A first reaction may be that the two must be connected; indeed when Joseph Needham （1959: 96 n. a 147） finds the same diagram in Bhaskara II （he gives no reference） and believes it to be found nowhere else he deems it "extremely probable that Bhaskara´s treatment derives from" Zhao Shuang´s commentary.

A connection between a Mesopotamian and a Chinese diagram is certainly not to be excluded *a priori*. There can be no doubt that the shared problem of the "hundred fowls" is really shared and thus that some kind of mathematics *did* travel. [①] The earliest known occurrence of this problem type is in the fifth-century *Zhang Qiujian Suanjing*（van Hée 1913） but soon it turns up not only with the same mathematical structure but also with shared parameters and dress （100 units of different prices and a total price of 100 – only the choice of species and their individual prices vary） in Carolingian Western Europe in India and in the Islamic World – see e. g. （Libbrecht 1974）; this cannot be imagined to be an accident.

We may add an observation elementary but to my knowledge not made before. The early Islamic Indian and Chinese occurrences speak of fowls the Carolingian *Propositiones ad ac-uendos iuvenes*（Folkerts 1978） has various dresses but none with fowls. This supports Jean Christianidis´ suggestion （1994: 240*f*） that the problem has developed from an early form represented in a Greek papyrus from the second century CE where the units are already 100

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[①] For the moment I shall pretend that the notion of "shared problem" （or shared mathematical knowledge） is unambiguous; this corresponds to such classical discussions of inter-cultural mathematical connections as （Hermelink 1978） and （Vogel 1983）. In the context of these discussions it was perhaps unproblematic to leave that semantic point to the understanding of the reader. We shall see below that it is not so for the present topic – but clarification is better postponed until we have our hands on material where it can be done concretely.

but the price 2500. [①] Once the more striking version 100/100 was invented  that was the one that spread east and west – but the fowls only eastward  for which reason this latter invention can be presumed to be a secondary accretion.

### Other possible interactions

The hundred fowls is an arithmetical riddle.  Another arithmetical riddle that *may* have travelled far is the one known in the cultures connected to the Mediterranean as the "purchase of a horse".  Like the "hundred fowls" it circulated with varied numerical parameters  but a typical example states that three men go to the market in order to buy a horse.  The first says that he has enough to pay the price if he can have half of the possession of the other two;  the second only needs one third  the last only one fourth of what the other two have.  The possession of each and the price of the horse is asked for.  Sometimes the price of the horse is given.

The problem seems to be hinted at in book I of Plato's *Republic* (333BC) . [②] In any case there can be no reasonable doubt that it turns up  undressed as pure–number problems  in Diophantos' *Arithmetica* I: 24–25 (Tannery 1893: I  56–69) ;  the preceding propositions 22–23  moreover  ask a question which  if dealing with a purchase  would make each participant ask for the fractions 1/3  1/4 and 1/5 respectively 1/3  1/4  1/5 and 1/6 of the possession not of all the others but of the one that precedes in a circle.

The latter type has an interesting parallel in Chapter VIII  problem 13 of the *Nine Chapters* [③]:

> Let us assume that with five families sharing a well  that what is missing for the two ropes of *Jia* (in order to reach the bottom of the well)  that is as one rope of *Yi*  that what is missing for the three ropes of *Yi*  that is as one rope of *Bing*  that is as one rope of *Bing*  that is as one rope of *Wu*  that what is missing for the six ropes of *Wu*  that is as one rope of *Jia*; and that if each gets the rope that is missing for him  all will reach the bottom of the well. One asks for the depth of the well and for the length of the ropes.

The mathematical structure of the problem is the nearly the same  including the characteristic attractive sequence of fractions.  However  one of $n$ ropes is spoken of instead of the fraction $1/n$ of the totality of the ropes of each (which avoids the need to cut the ropes) ; moreover  the request is made to the following participant  not to the predecessor in the circle.

Problems 3 and 12 (*ibid.* p. 625  641) are determinate but otherwise similar in structure.  No. 3 speaks of 2 bundles of millet of high quality  3 of medium quality and 4 of low

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① Text in (Winter 1936: 39) .

② Trans. (Shorey 1930: I  332*f*) Socrates asks when one needs an expert; and as usually he answers himself: for example when you go to the market to buy or sell a horse in common. Since horses did not serve in agriculture but only for military purposes  they would never be bought in common in real life. (I owe the discovery of the Platonic passage to the late Benno Artmann) .

③ My English from the French of (Chemla&Guo 2004: 643) .

quality   and in similar combinations they are to produce 1 *dou*; no. 12 deals with the hauling capacity of horses of different strength. Problems 14 and 15 (*ibid*. pp. 645  647) are sophisti-cated variants – we shall return to them.

The coincidences may seem striking – but are they evidence *of connection* or *of parallel ex-periences* of fascination? *If* the far from obvious dress had also been shared   as in the case of the hundred fowls   then connection would seem next to certain. Since it is not   we cannot decide on the basis of these problems alone.

However   problem 10 (*ibid*. p. 639) supports the connection hypothesis. It presents us with something like a two-participant version of the problems we have just examined. Two per-sons own money; if *Jia* gets half of what *Yi* possesses   he will have 50 coins; and if *Yi* gets 2/3 of what *Jia* possesses   he will have as much.

With only two participants   there is no difference between Diophantos's two types. We notice   firstly   that here fractions and not "one out of $n$" are spoken about; secondly   that the dress is the familiar "give-and-take" type.

This dress is used for a slightly different mathematical structure in problems from late Med-iterranean Antiquity (and later). In Book XIV of the *Greek Anthology* (Paton 1916: V  105) no. 145 runs

*A*. Give me ten minas   and I become three times as much as you. *B*. And if I get the same from you

I am four times as much as you.

No. 146 uses different numerical parameters (two minas   twice   four times) but is otherwise identical. Prop. XV of Diophantos's *Arithmetica* I (Tannery 1893: I  36f) is an undressed ver-sion of the same problem type.

On the other hand   Fibonacci's first example of a "purchase of a horse" (Boncompagni 1857: 228) has the same mathematical structure as the Chinese give-and-take problem   apart from being indeterminate. In Fibonacci's problem   the first man asks for 1/3 of the possession of the second   while the second asks for 1/4 of what the first has. In both cases   each will have enough to buy the horse (whence the same amount).

Again   this coincidence in isolation suggests but does not prove a connection. However the alternative explanation here cannot be fascination with interesting numbers but only acci-dent. If we take together all the problems we have looked at   independent invention in the two areas becomes unlikely – not least because the case of the "hundred fowls" provides us with firm evidence that transmission could and sometimes did take place.

But what exactly can have been transmitted? All these problems from the *Nine Chapters* come from Chapter VIII   and all are used to train the *fangcheng* method. Diophantos's methods are quite different – and no closer are variants of the "Bloom of Thymarides" (Heath 1921: I. 94–96)   which may have been used already around or before Plato's times to solve similar problems.

In connection with the "hundred fowls"   Ulrich Libbrecht(1974: 313) points out that

> This implies that several mathematical problems were transmitted only as questions without any method as we can clearly state in Alcuin's work ( the *Propositiones ad acuendos iuvenes* /JH) ; in different places methods were developed – wrong or right – to solve these problems. Perhaps they were considered more as games than as serious problems as we can prove from several Chinese and European works.

The same is clearly the case here. In terms I have used in ( Høyrup 1990) ( not knowing by then about Libbrecht's observation) the problems have circulated as "subscientific" mathematics more precisely as professional riddles belonging to an environment of mathematical practitioners; once taken up by groups which in some way can be characterized as scholarly mathematicians ① these developed their own ways to deal with them and in some cases they expanded the range of questions these methods could be applied to. Since the riddles functioned precisely as *riddles* in the community of practitioners ( in anthropological parlance as *neck riddles* – who is not able to solve them is "not one of us") it is not even certain that the practitioners always had a *mathematical* method for solving them – a riddle asks for an answer not for a calculation or a logical derivation and a guess followed by a verification may have been enough. ②

That is where problems no. 14 and 15 of Chapter VIII of the *Nine Chapters* come in. No. 14 deals with groups of unit fields of millet with different yields – say $2A$ $3B$ $4C$ and $5D$. But this time $2A + B + C = 3B + C + D = 4C + D + A = 5D + A + B = 1$ *dou*. This is too complex to present a nice recreational riddle – to keep track of it without material support would be difficult. No. 15 deals with three groups of bundles of millet of different weights but speaks of differences instead of sums – in symbols $2A - B = 3B - C = 4D - A = 1$ *dan*.

Such extensions of the range of "recreational" riddles by variation and systematization at the hands of scholarly mathematicians are a common occurrence in history from Old Babylonian times to Pedro Nuñez and beyond. Diophantos in *Arithmetica* I replaces variation by generalization – but his choice of numerical examples betrays the recreational starting point.

At times however "scholarly mathematicians" have made a further step and used the recreational material as the starting point or inspiration for the creation of a whole mathematical discipline. That is the way Old Babylonian second-and third-degree "algebra" was generated. ③ In the whole corpus there is not a single second or third-degree problem derived from a genuine practical question that might present itself to a Babylonian scribal calculator. We should not be

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① That is people who are engaged in or linked to a school-based ( as opposed to an apprenticeship-based) educational system and who in that connection shape and transmit mathematical knowledge.

② This is precisely what Abū Kāmil reproaches those who took pleasure in the "hundred fowls" in his times and surroundings – it was "a particular type of calculation circulating among high-ranking and lowly people among scholars and among the uneducated at which they rejoice and which they find new and beautiful; one asks the other and he is then given an approximate and only assumed answer they know neither principle nor rule in the matter" – my translation from ( Suter 1910: 100).

③ See for instance ( Høyrup 2001).

misled by the fact that the *entities* occurring as "unknowns" in the problems would be familiar to him – dimensions of fields and excavations, prices, manpower, etc. In genuine surveying, one would (for instance) never have to determine the sides of a rectangular field from its area and the sum of the sides; but exactly such recreational riddles served as basis for the new discipline.

If we now consider Chapter VIII of the *Nine Chapters* in its integrity, the parallel becomes obvious. Although the *Nine Chapters* on the whole teach administrators' mathematics, Chapter VIII does not present us with a single instance of this. True, *the entities* that occur would again (mostly) be of the kind dealt with by calculating bureaucrats (the combined ropes hardly); but *the problems* would never turn up in their offices. Moreover, the book as a whole is a theoretical unity. Since the topic is absent from the *Suàn shù shū* (Cullen 2004: 6; *id*. 2007: 29; Dauben 2008: 97  131), Chapter VIII can be assumed to be the outcome of recent systematic establishment of a well-defined mathematical field – inspired in all likelihood by select recreational problems as received within the existing mathematical culture, with its techniques, approaches and metamathematical ideals; administrative mathematics *per se* would not lend itself adequately to that role.

All in all, Chapter VIII and its Mediterranean kin thus appears to present us with all the facets involved in questions about transmission:

1. Transmission of *problems* as riddles from an unidentified *somewhere* to both the classical Mediterranean area and Han China (and other locations). [1]

2. Local creation of adequate *methods*.

3. A creation of a mathematical *discipline* on this foundation in China, in a process that is parallel to what can be seen in Old Babylonian mathematics. This parallel was based on shared sociological conditions and certainly did not involve any kind of transmission of metamathematical ideals. [2]

Problems about "combined works" present themselves easily in all cultures of scribal mathematical administration, and there are basically only two reasonable ways to solve them; [3] neither the occurrence of such problems in different places nor a shared way to proceed can thus be

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[1] *Somewhere* must be emphasized. In questions of this kind it is misleading to take for granted that the ultimate source must be one of the literate "nationally" defined high cultures we know about – "the Chinese", "the Greeks", "the Indians", etc. "Proletarians have no fatherland", it was claimed – until the experience of the First World War proved the opposite in France and Germany. Often, merchants and technicians (even highly qualified technicians à la Wernher von Braun) still have none.

[2] Those of the Old Babylonian school had died with the school itself around 1600 BCE, more or less at the time of the earliest oracle bones. Even if that had not been the case, however, transmission could be safely excluded – institutional ideals can only be exported *with understanding* (whence with efficiency) if the institution itself is exported.

[3] If *A* can complete the task in 3 days and *B* in four, then each day *A* completes 1/3 and *B* 1/4, etc. Alternatively, in $3 \times 4$ days *A* will complete it 4 times, and *B* 3 times, the two together thus 7 times, etc. If the numbers had not been mutually prime, the latter method can be found in the variant that the smallest common multiple is chosen, but apart from that sources I have looked at know no other ways. Obviously they are algebraically equivalent.

taken as evidence of transmission.  An unlikely dress can  however (as in the case of the "hun-dred fowls").

Such a case is present in Chapter VI  problem 6 of the *Nine Chapters* (Chemla & Guo 2004: 541).  Here  a pool is filled from five streams.  As it is  filling is a preferred dress for such problems in the *Greek Anthology* XIV – thus no. 7  130–133  135 (Paton 1916:  V  31  97  99).  Shared transmission from somewhere is thus likely – but since this is no *favourite* dress of the problem type in the *Nine Chapters* (other instances – no. 22  23 and 25 – really concern working rates)  an isolated borrowed recreational problem may simply have been in-serted in an adequate place of the *Nine Chapters*  the writer having recognized an already famil-iar type. [1]

All of these cases of credible transmission  from the "hundred fowls" onward  are number problems;  they are of the kind that would allow an accountant or a travelling merchant to show his mathematical proficiency.  Since accountants are likely to stay more or less in their place  travelling merchants constitute the plausible carrying community for these riddles;  at an earlier occasion (Høyrup 1990: 74) I have spoken of them as the "Silk Route group".

## Seleucid and Demotic Geometry

We started with a suggestive geometric diagram  and then shifted focus to the possible transmis-sion of arithmetical riddles.  Let us return to geometry.

Our initial diagram is too isolated to be worth pursuing.  More intriguing is the geometry of Chapter IX of the *Nine Chapters* in relation to certain geometric problems from Seleucid Mesopo-tamia (third to second century BCE) and Hellenistic-Demotic Egypt.

The Seleucid problems in question (mainly) deal with rectangles with a diagonal.  They have a family relationship with the Old Babylonian so-called "algebra" – apparently not by di-rect descent but via shared borrowing from the riddles of practical surveyors. [2]

Most of the problems in question are known from the tablet BM 34568 [3] undated but probably

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[1]   No. 21 and 22 are in the dress of travel times  which also shows up elsewhere in later times. No. 27 and 28  of structure "box problems"  are in the dress of repeated taxation  also familiar from India and elsewhere in the later first and early second millennium. Chapter VI thus serves  it seems  as a receptacle for several widely circulating rec-reational problems for which it presents the earliest written evidence (much as Diophantos´s *Arithmetica* I).

[2]   The evidence for this is in part linguistic  in part it has to do with strong reduction (followed by expansion) at the level of mathematical substance. See (Høyrup 2002a: 389–399).

  The kinship does not imply that the Seleucid problems represent an "algebra". Whether the Old Babylonian technique does so is a matter of how we define algebra  but no reasonable definition will cover the Seleucid rectangle problems – they do not serve any kind of representation  and the method is purely synthetic.

[3]   (Neugebauer 1935: III  14–19)  partial edition  "conformal" translation and commentary in (Høyrup 2002a: 392–399).

from the later third or earlier second century BCE. Its problems can be described as follows: [1]

(1)  $l = 4$   $w = 3$; $d$ is found as $\frac{1}{2}l + w$ – first formulated as a general rule   next done on the actual example.

(2)  $l = 4$   $d = 5$; $w$ is found as $\sqrt{d^2 - l^2}$.

(3)  $d + l = 9$   $w = 3$; $l$ is found as $\dfrac{1/2 \cdot (\overline{d+l}^2 - w^2)}{d+l}$   $d$ as $(d+l) - l$.

(4)  $d + w = 8$   $l = 4$; solution corresponding to (3).

(5)  $l = 60$   $w = 32$; $d$ is found as $\sqrt{l^2 + w^2} = 68$.

(6)  $l = 60$   $w = 32$; $A$ is found as $l \cdot w$.

(7)  $l = 60$   $w = 25$; $d$ is found as $\sqrt{l^2 + w^2} = 65$.

(8)  $l = 60$   $w = 25$; $A$ is found as $l \cdot w$.

(9)  $l + w = 14$   $A = 48$; $\langle l - w \rangle$ is found as $\sqrt{(l+w)^2 - 4A} = 2$   $w$ as $\frac{1}{2} \cdot (\overline{l+w} - \langle l-w \rangle)$  and $l$ finally as $w + \langle l-w \rangle$.

(10)  $l + w = 23$   $d = 17$; $\langle 2A \rangle$ is found as $(\overline{l+w}^2 - d^2) = 240$   $\langle l - w \rangle$ next as $\sqrt{(l+w)^2 - 4A} = 7$ – whence $l$ and $w$ follow as in (9).

(11)  $d + l = 50$   $w = 20$; solved as (3)   $l = 21$   $d = 29$.

(12)  deals with a reed leaned against a wall   cf. imminently; a corresponding rectangle problem would be $d - l = 3$   $w = 9$; $d$ is found as $\dfrac{1/2 \cdot (w^2 + \overline{d-l}^2)}{d-l} = 15$   $l$ as $\sqrt{d^2 - w^2} = 12$.

(13)  $d + l = 9$   $d + w = 8$; $\langle l + w + d \rangle$ is found as $\sqrt{(d+l)^2 + (d+w)^2 - 1} = 12$ where 1 obviously stands for $(l-w)^2 = (\overline{d+l} - \overline{d+w})^2$; next $w$ is found as $\langle l+w+d \rangle - (d+l) = 3$   $d$ as $(d+w) - w$   and $l$ as $(d+l) - d$.

(14)  $l + w + d = 70$   $A = 420$; $d$ is found as $\dfrac{1/2 \cdot (\overline{l+w+d}^2 - 2A)}{l+w+d} = 29$.

(15)  $l - w = 7$   $A = 120$; $\langle l + w \rangle$ is found as $\sqrt{(l-w)^2 + 4A} = 23$   $w$ as $\frac{1}{2} \cdot (\langle l+w \rangle - \overline{l-w}) = 8$   $l$ as $w + (l-w)$.

(16)  A cup weighing 1 mina is composed of gold and copper in ratio $1 : 9$. [2]

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[1]  $l$ stands for the length   $w$ for the width   $d$ for the diagonal and $A$ for the area of a rectangle. The sexagesimal place value numbers are transcribed into Arabic numerals. Entities that are found but not named in the text are identified in $\langle\ \rangle$. With minor changes   I draw the list from (Høyrup 2002b: 13$f$).

[2]  Obviously an intruder   which however shows that at least some problems from the "Silk Road group" were already known in Mesopotamia at the time. In the present context there is no reason to discuss this connection in depth. Similar connections *may* be in play in the seemingly aberrant problem 1   extensively discussed   but with a different aim   in (Gonçalves 2008).

（17）　$l + w + d = 12$　$A = 12$; solved as （14）　$d = 5$.

（18）　$l + w + d = 60$　$A = 300$; not followed by a solution but by a rule formulated in general terms and cor-responding to （14） and （17）.

（19）　$l + d = 45$　$w + d = 40$; again  a general rule is given which follows （13）.

No. 2 is obviously an application of what I prefer to call the "Pythagorean rule"  no theorem being involved; it corresponds to knowledge that was amply around in Old Babylonian times. No. 10 is not iden-tical with the Old Babylonian problem which I referred to initially （Db$_2$-146）; but it is closely related and solvable by means of the same diagram.

All the others （disregarding here and in what follows the intruders no. 1 and no. 16） represent innovations within the surveyor´s riddle tradition – when not in question then in method （no. 6 and 8 obviously present old knowledge  which however had been taught at an elementary level）. No. 12 de-serves particular discussion. It deals with a reed of length $d$ first standing vertically against a wall  next in a slanted position  in which the top descends to height $l$ （descending thus $d - l$）; at the same time  the foot moves a distance $w$ away from the wall.
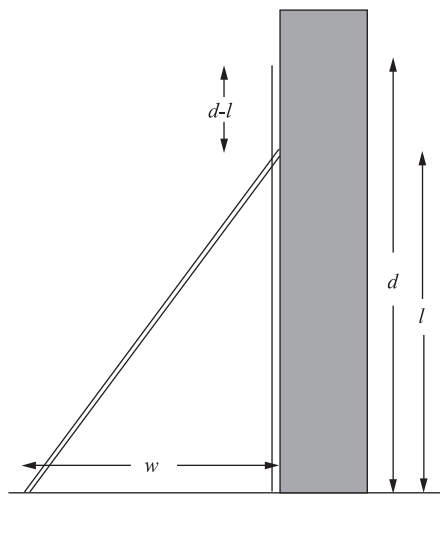


Fig. 2　The reed leaned against the wall.

In the Old Babylonian text BM 85196 we find a similar dress  but there $d$ and $w$ are given. To find $l$ thus requires nothing but direct application of the Pythagorean rule （similarly to no. 2 here）. In the present case  instead  as stated  the descent $d - l$ is given together with $w$.

It is possible to find plausible geometric explanations of the procedures used to solve all the "new" problems – see （Høyrup 2002b: 13–18）; that  however  is of no interest in the present connection （in particular  of course  because we have no certainty that these explana-tions were those which the ancient calculators thought of）.

BM 34568 is not our only source for this kind of rectangle problems.  Firstly  the Seleucid text AO 6484[①] contains a rectangle problem of the same type as nos. 14  17 and 18 of BM 34568. Secondly  the Demotic papyrus P. Cairo J. E. 89127–30  89137–43 from the third century BCE[②] contains eight problems about the reed leaned against the wall – three of the easy Old Babylonian type where $d$ and $w$ are given  three of the equally simple type where $d$ and $d - l$ （and therefore easily $l$） are given; and two  finally  where $d - l$ and $w$ are given  as in BM 34568 no. 12. Though with different numerical parameters  moreover  two of its problems

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① Ed. （Neugebauer 1935: Ⅰ 96–99）; early second century BCE.

② （Parker 1972: 13–53）  with summary *ibid*. pp. 3*f*.

coincide with that of the Old Babylonian text $Db_2$-146; they are thus closely related to BM 34568 no. 10.

There can be no doubt that the ultimate source for this whole cluster of geometric problems is Mesopotamia or at least West Asia – Pharaonic mathematics contains nothing similar. It is also easy to pinpoint a professional community that could transmit it: For half a millennium  Assyrian  Persian and Macedonian military surveyors and tax collectors (even those of the Macedonians no doubt trained in the Near Eastern tradition) had walked up and down Egypt. [1]

## *Travelling geometry – travelling how far*?

It is less easy to identify the channels through which these problems came to be adopted into Jaina mathematics  as they certainly were (Høyrup 2004). Our evidence is constituted by Mahāvīra´s *Ganita-sāra-sangraha* from the ninth century CE  but it is obvious from Mahāvīra´s words that by then these problems were considered old  native and venerable by the Jainas. It is also highly plausible that what reached them had already been digested and somewhat transformed by a broader Mediterranean community.

Given that they reached India  could they also have inspired China´s mathematical bureaucrats  or at least the author of Chapter IX of the *Nine Chapters* (which has no more to do with real administrative tasks than Chapter VIII)?

At a first glance  problems 6 to 12 and 24 might suggest so. In that case  however  the inspiration has certainly been digested – the basic topic of Chapter IX is the right *triangle*  and the Seleucid-Demotic problems deal with rectangles. [2]Moreover  the Chinese variant of the reed against the wall (no. 8 – the only one where the dress is suggestive) compares the slanted and the *horizontal* position of a pole.

If we express the Chinese problems in the same symbolic form as used for BM 34568  we get the following: [3]

(6)　$d - l = 1$　$w = 5$ (mathematically an analogue of the Seleucid reed problem BM 34568 no. 12  but dealing with a reed in a pond  in vertical and slanted position). $l$ is found as $\dfrac{w^2 - (d-l)^2}{2 \cdot (d-l)}$  whence $d$  where BM 34568 finds $d$ as $\dfrac{1/2 \cdot (w^2 + (d-l)^2)}{d-l}$ and next $l$. We observe that the Chinese procedure does not halve before dividing by $(d-l)$  which suggests that a geometric justification  *if* once present  had been forgotten.

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[1]　Macedonians excepted  this explains why Greek authors  from Herodotos onward  could believe the Egyptians to have invented geometric techniques which we now recognize as Mesopotamian.

[2]　Problem 11 in the *Nine Chapters* then *applies* the triangle techniques to a problem about a rectangular door  while the Seleucid text applies its rectangle techniques to the slanted pole  a triangle problem. Both writers obviously understood the connection  but their choice of different basic models is still to be taken note of.

[3]　Once more I borrow (this time more freely) from (Høyrup 2002b).

(7)   $d - l = 3$   $w = 8$. The mathematical structure of the problem is the same   but the concrete dress wholly other. The solution proceeds differently than everything Seleu‐cid‐Demotic: $\langle d + l \rangle$ is found as $\dfrac{w^2}{d - l} = \dfrac{(d + l) \cdot (d - l)}{d - l}$   and $d$ as

$$\frac{1}{2}(\langle d + l \rangle + d - l).$$

(8)   $d - l = 1$   $w = 10$. Same mathematical structure and same procedure as no. 7 – but the dress is now a pole first leaning against a wall and then sliding down to horizon‐tal position.

(9)   Another variation of no. 7.

(10)   Yet another variation of no. 7.

(11)   $d = 100$   $l - w = 68$. $\left(\dfrac{l + w}{2}\right)$ is found as $\sqrt{\dfrac{d^2 - 2 \cdot \left(\dfrac{l - w}{2}\right)^2}{2}}$ and in the following step $w$ as

$\left(\dfrac{l + w}{2}\right) - \dfrac{l - w}{2}$. Not Seleucid – Demotic in style with its use of aver‐age and deviation – nor however similar in detail to anything from the older Mesopotamian tradition where these quantities were funda‐



**Fig. 3   The possible geometric basis for problem IX. 24 of the *Nine Chapters*.**

mental   however much the immediate topic is a rectangle and no triangle.

(12)   $d + l = 10$   $w = 3$. $\langle d - l \rangle$ is found as $w^2/(d + l)$   and $l$ as

$\dfrac{1}{2}(d + l + d - l)$ – once more different from the Seleucid calculation.

(24) $d - l = 2$   $d - w = 4$. The solution presupposes that
☐$(d - d - l - d - w) = 2(d - l) \cdot (d - w)$. ① The problem type is not found in BM 34568   but the solutions can be argued from a diagram that can also be used to solve problem 13 of that text   $d + l = \alpha$   $d + w = \beta$. In the present case the full square ☐$(d)$ must equal the sum of the squares ☐$(l)$ and ☐$(w)$; there‐fore   the overlap $S = $☐$(d - d - l - d - w)$ must equal the area which they do not cover   that is   $2R = 2$☐$(d - l \ d - w)$.

All in all   the similarities boil down to the mathematical structures underlying *the questions*. The dresses are generally quite different   and the procedures used to obtain the solutions are
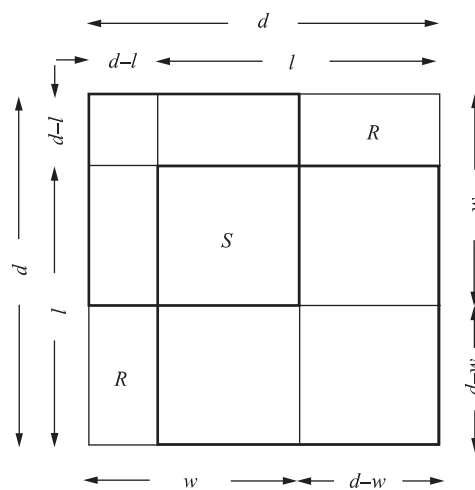
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①   ☐$(a)$ stands for the square on $a$   more precisely its area ☐$(a \ b)$ for the rectangle contained by a and b.

also others  often as different in character as the subject allows. In summary  no decisive inter-nal evidence speaks in favour of transmission – only problem 7 *could* be an accidental intruder that has been inserted in the adequate place  as the filling problem 6 in Chapter VI.

External evidence also seems unfavourable to the transmission thesis: arithmetical riddles might be carried along the Silk Road network by travelling merchants and exchanged as camp fire fun or challenges. But where can we find likely carriers of geometrical questions?

Admittedly  *some* geometrical knowledge – albeit fairly useful area measurement and no riddle – appears to have travelled; when and from where we do not know  nor carried by whom. Problems 35–36 of Chapter I of the *Nine Chapters*( Chemla & Guo 2004: 190*f*) deter-mine the area of a circular segment with chord $c$ and arrow $h$ as $\dfrac{hc + h^2}{2}$ whereas problem 36 of the Demotic papyrus Cairo J. E. 89127–30  89137–43 ( Parker 1972: 45) takes it to be $\dfrac{h + c}{2} \cdot h$. It is not exactly the same formula  but on the other hand the rule is far from intuitively self-evident. It is certainly true for the semi-circle if we take the perimeter to be thrice the diameter ( as both sources do) – that was seen by Heron ( Schöne 1903: 74*f*) as well as Liu Hui( Chem-la & Guo 2004: 193)  both of whom also point out that for other situations it provides nothing but an approximation. But in the case of a semicircle we have that $c = 2h$  and the formula re-duces to $\dfrac{3}{2}h^2$; it is not obvious how this should be generalized as done in the Chinese as well as the Demotic text. Independent invention therefore seems unlikely. ( As carriers  should we think of surveyors or military experts being taken prisoners or shifting their loyalties?) .

There is a fundamental difference  however. In this case the formula has travelled together with the problem to which it is linked. If diffusion should be involved in the problems of Chap-ter IX  all that was transmitted was the mathematical structure of the problems  neither the actual questions to which they correspond nor the formulas used to solve them. This could only be carried by people who would recognize as essential an abstract mathematical structure behind the problems – that is  by people who must be characterized as *mathematicians*. They were hardly at hand for the task.

All in all  Chapter IX of the *Nine Chapters*( of which problems 6–13 + 24 only form a sub-set) is therefore likely to be just as much an original Han creation as Chapter VIII – but with the difference that the underlying inspiration must be sought in local geometric practice  and not in the practice or riddles of any transnational professional community. The *Gnomon of the Zhou*  on which Zhao Shuang was certainly not the first to work  would suggests the practice of astral science. Since Chinese astral science is a field about which I am quite ignorant  I shall not pursue this suggestion  which is anyhow outside my topic.

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